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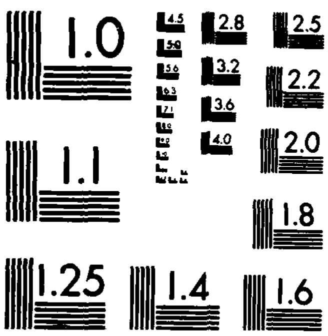

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NSWC TR 86-53

EARTH TIDE ALGORITHMS FOR THE OMNIS COMPUTER PROGRAM SYSTEM

BY WALTER J. GROEGER
STRATEGIC SYSTEMS DEPARTMENT

APRIL 1986

NAVAL SURFACE WEAPONS CENTER
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report presents five computer algorithms that jointly specify the gravitational action by which the tidal redistributions of the Earth's masses perturb satellite motion. Covered are the perturbing effects of the lunar and solar atmospheric tide bulges as well as those of the ocean tides and the solid Earth tide. The lunar air tide algorithm is restricted to the dominant, semidiurnal tide component. The atmospheric tide routine associated with the Sun combines the effects of solar heating and solar mass attraction. It includes the semidiurnal and diurnal terms. The model for the ocean tide contains the semidiurnal lunar term and 10 additional tide constituents. It also features the effect on the solid Earth tide of the hydrostatic loading by the ocean tide. The algorithm for the solid Earth tide is based on an abbbreviated form of the Wahr model outlined in the final version of the MERIT Standards Document. It includes a variety of spectral modes associated with the long-periodic and the semidiurnal tides. The fifth computer routine is a simplified version of the fourth and is provided for use during computer program verification. All computer algorithms express the tidal modifications of the perturbing acceleration experienced by the satellite in terms (Over)

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11. (Cont.) OMNIS COMPUTER PROGRAM SYSTEM

19. (Cont.) of corrections to the expansion coefficients of the Earth's gravitational potential. Time enters the model by means of closed-form expressions for the astronomical fundamental arguments adapted from the MERIT Standards nutation model.

FOREWORD

This document contains the analytical formulation of five computer algorithms specifying the gravitational action by which the tidal redistribution of the Earth's masses perturbs satellite motion. The five new computer routines were developed for use with the OMNIS system of satellite geodesy computer programs under MIPR No. HM0027-86-C047.

The work was performed in the Space and Surface Systems Division. This report has been reviewed by R. L. Kulp, Head, Space and Ocean Geodesy Branch; and C. W. Duke, Jr., Head, Space and Surface Systems Division.

Approved by:

D. B. Colby

D.B. COLBY, Head
Strategic Systems Department

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ILLUSTRATIONS

INTRODUCTION

The following five algorithms formulate the perturbing potentials associated with the four Earth tides[1,2,3,4,5] in terms of linear combinations of the eigenfunctions U_{nm} and V_{nm} (solid harmonics) used to express the main gravitational field in OMNIS.[6] They make it possible to calculate the gravitational effect on the satellite orbit of the mass redistribution caused by the tides by simply adding the generally time-dependent expansion coefficients of the expressions for the tidal perturbing potentials as increments to the expansion coefficients C_{nm} and S_{nm} of the main gravitational potential field. Also, by processing the tidal increments to the main field coefficients along with the latter, the computation of the second derivatives with respect to the field-point coordinates necessary to include the contribution of the Earth tides to the variational equations occurring in connection with the orbit determination becomes an entirely automatic part of the contribution by the main gravitational field parameters to the variational equations.

The first and second algorithms specify the lunar atmospheric tide and the air tides produced by the combined gravitational and thermal effects of the Sun, respectively. These algorithms had originally been formulated[1,7,8] directly as perturbing accelerations. They were, for the purposes of OMNIS, rewritten in terms of the required potential expansion coefficient increments. Only minor changes were needed to adapt the CELEST Extended Ocean Tide[2] to OMNIS as its third Earth tide routine. The desired expansion coefficient increments were already part of the CELEST version. The fourth OMNIS tide algorithm accounts for the solid Earth tide as specified by the MERIT campaign document.[3] This particular

tide formulation was readily expressed in the form necessary for OMNIS. And, finally, the fifth routine is a reformulation of the familiar, elementary, solid Earth tide that was part of earlier orbit integration routines contained in computer programs such as CELEST,[9] TERRA,[10] and ASTRO.[11] It is included here because some users of OMNIS may occasionally want to activate it.

DATA ASSOCIATED WITH TIME AND THE FUNDAMENTAL ASTRONOMICAL ARGUMENTS

INPUT DATA

- Y year (four digits)
- D day number (number of day in year Y)
- t^* Universal Time in seconds (s)

ALGORITHM

Using double precision throughout, evaluate the time instant T of the time line under consideration, expressed in Julian Centuries of 36525 days of 86400 s of Dynamical Time elapsed since Epoch J2000.0 (= JD 2451545.0 = 2000 January 1.5)

$$M = 1 \tag{101}$$

$$\begin{aligned} N_\Sigma = {} & 367 * Y - 7 * (Y + (M + 9)/12)/4 \\ & - 3 * ((Y + (M - 9)/7)/100 + 1)/4 \\ & + 275 * M/9 + 1721029 + D \end{aligned} \tag{102}$$

$$\begin{aligned} \mathrm{MJD0} &= N_\Sigma - 0.5 - 2400000 \\ &= \text{Modified Julian Date at beginning of day D, UT} \end{aligned} \tag{103}$$

$$T_0 = \frac{\mathrm{MJD0} - 51545}{36525} \tag{104}$$

$$T = T_0 + \frac{t^*}{86400 * 36525} \tag{105}$$

Note that the equation for N_Σ is a FORTRAN expression. When evaluating it outside of a FORTRAN computer program, the rules of FORTRAN integer arithmetic must be observed. This expression computes the Julian day number corresponding to a time instant 43200 s into (noon UT of) the day D, during month M and year Y. It is valid for all A.D. years. Y, M, and D indicate the Gregorian calendar date. Evaluate also

$$\begin{aligned}\ell &= 134.96298139 + 477198.867398056\ T \\ &\quad + 0.008697222\ T^2 + 0.000017778\ T^3\end{aligned} \tag{106}$$

$$\begin{aligned}\ell' &= 357.527723333 + 35999.050340000\ T \\ &\quad - 0.000160278\ T^2 - 0.000003333\ T^3\end{aligned} \tag{107}$$

$$\begin{aligned}F &= 93.271910278 + 483202.017538056\ T \\ &\quad - 0.003682500\ T^2 + 0.000003056\ T^3\end{aligned} \tag{108}$$

$$\begin{aligned}D &= 297.850363056 + 445267.111480000\ T \\ &\quad - 0.00191417\ T^2 + 0.00000528\ T^3\end{aligned} \tag{109}$$

$$\begin{aligned}\Omega &= 125.0445222 - 1934.13626083\ T \\ &\quad + 0.00207083\ T^2 + 0.00000222\ T^3\end{aligned} \tag{110}$$

All five quantities result in decimal fractions of degrees. Further evaluate the lunar time angle, in degrees, reckoned from lower transit

$$\tau = \theta_g + 180° - s \tag{111}$$

the Moon's mean longitude, in degrees

$$s = F + \Omega \tag{112}$$

The Sun's mean longitude, in degrees

$$h = s - D \tag{113}$$

the longitude, in degrees, of the Moon's mean perigee

$$p = s - \ell \tag{114}$$

the negative longitude, in degrees, of the Moon's mean node

$$N' = -\Omega \tag{115}$$

and the longitude, in degrees, of the Sun's mean perigee

$$P_1 = s - D - \ell' \tag{116}$$

where

θ_g is the Greenwich mean sidereal time, in degrees.

$$\theta_g = 100.4606184 + 36000.7700537\, T_0 + 0.000387933\, T_0^2 + \frac{1.002737909}{3600} \cdot 15 \cdot t^* \tag{117}$$

Reduce the quantities calculated by Equations 106 through 117 Modulo 360.

SPECIFICATION OF EIGENFUNCTIONS

The following equations are *not* part of the algorithm. They are statements intended to clarify what precisely is meant whenever the term "eigenfunction" appears in the present coding request.

$$U_n^m \equiv \mu \frac{R^n}{r^{n+1}} P_n^m(\sin\theta) \cos m\lambda \tag{201}$$

$$V_n^m \equiv \mu \frac{R^n}{r^{n+1}} P_n^m(\sin\theta) \sin m\lambda \tag{202}$$

where

μ = Earth's gravitational parameter (= gravitational constant G times Earth's mass)

R = semimajor axis of reference ellipsoid (Earth's "radius")

r = geocentric distance of field point

θ = field point geocentric latitude

λ = field point longitude in Earth-fixed frame

$p_n^m(x)$ = Legendre Polynomials (normalization as stated in "JAHNKE-EMDE")

The relationship of these eigenfunctions with the gravitational expansion coefficients $C_{n,m}$ and $S_{n,m}$ and with the Earth's main gravitational potential, U, is

$$U = \sum_{n,m} (C_{n,m} U_n^m + S_{n,m} V_n^m) \tag{203}$$

The following computer program segments correspond to the individual tides and produce increments $\Delta C_{n,m}$ and $\Delta S_{n,m}$ to the $C_{n,m}$ and $S_{n,m}$. These increments reflect the effect of the particular Earth tide on the satellite motion and on the variational equations.

Note that the above eigenfunctions, potential expansion coefficients, and increments to the latter expansion coefficients are those quantities that are normally referred to as "unnormalized" eigenfunctions and expansion coefficients to be distinguished from the corresponding "normalized" quantities.* Note that the MERIT Campaign Document uses normalized quantities for its version of the solid Earth tide.

*If an elaboration of the terms "normalized" and "unnormalized" is needed, see NSWC TR-2223 A Geoid Height and Gravity Anomaly Computer Program For Geodetic Parameter Solutions, W. Groeger, among other references.

As far as units and dimensions are concerned, it is assumed that for the purposes of formulating the Earth's main gravitational field, μ, R, and r will be expressed in terms of km and s (μ = 398600.5 km^3 s^{-2}). Accordingly, all C and S increments occurring below will be equipped with numerical factors that ensure the values calculated from these expressions will be compatible with the expansion coefficients of the main gravitational potential field.

CONTRIBUTIONS TO THE GRAVITATIONAL POTENTIAL EXPANSION COEFFICIENTS BY THE LUNAR ATMOSPHERIC TIDE

INPUT DATA

A_2 = a constant associated with the physics of the lunar air tide. Suggested value A_2 = 0.564 kg m^{-2}

R = any reasonable value for the mean "radius" of the Earth (semimajor axis of the earth's reference ellipsoid), in km. Default value R = 6378.140 km

G = gravitational constant in m, kg, and s. Suggested value G = 6.6732E-11 m^3 kg^{-1} s^{-2}

μ = Earth's gravitational parameter. Suggested value μ = 398600.5 km^3 s^{-2}

Y = year

D = day

t* = Universal Time in s

s(T) and h(T) = fundamental astronomical parameters

ALGORITHM

$$a = A_2 \, G \, R \, \frac{5\pi^2}{64} \tag{301}$$

$$\ell = a/48 \tag{302}$$

$$t^{**} = \frac{360}{86400} \, t^* \tag{303}$$

$$v = s(T) - h(T) \tag{304}$$

$$\Gamma = 2(t^{**} - v) - 15° \tag{305}$$

The increments to the gravitational potential expansion coefficients, due to the lunar air tide, are

$$\Delta C_{2,2} = + \frac{aR}{\mu} 10^{-3} \cos \Gamma \tag{306}$$

$$\Delta S_{2,2} = - \frac{aR}{\mu} 10^{-3} \sin \Gamma \tag{307}$$

$$\Delta C_{4,2} = - \frac{bR}{\mu} 10^{-3} \cos \Gamma \tag{308}$$

$$\Delta S_{4,2} = + \frac{bR}{\mu} 10^{-3} \sin \Gamma \tag{309}$$

CONTRIBUTIONS TO THE GRAVITATIONAL POTENTIAL EXPANSION COEFFICIENTS BY THE SOLAR ATMOSPHERIC TIDE

INPUT DATA

B_1 = a constant associated with the physics of the solar air tide. Suggested value $B_1 = 6$ kg m^{-2}

B_2 = a constant associated with the physics of the solar air tide. Suggested value $B_2 = 11.9$ kg m^{-2}

R = semimajor axis of Earth's reference ellipsoid, in kilometers. Default value R = 6378.140 km

G = gravitational constant in m, kg, and s. Suggested value G = 6.6732E-11 m^3 kg^{-1} s^{-2}

μ = Earth's gravitational parameter. Suggested value μ = 398600.5 km^3 s^{-2}

t^* = Universal Time in s

ALGORITHM

$$a_1 = B_1 \frac{8\pi GR}{105} \qquad (401)$$

$$a_2 = B_2 \frac{5\pi^2 GR}{64} \qquad (402)$$

$$a_3 = B_2 \frac{5\pi^2 GR}{3072} = a_2/48 \qquad (403)$$

The increments to the gravitational potential expansion coefficients, due to the solar air tide, are

$$\Delta C_{3,1} = - \frac{a_1 R}{\mu} 10^{-3} \cos \left(\frac{360}{86400} t^* - 78°\right) \qquad (404)$$

$$\Delta S_{3,1} = + \frac{a_1 R}{\mu} 10^{-3} \sin \left(\frac{360}{86400} t^* - 78°\right) \qquad (405)$$

$$\Delta C_{2,2} = + \frac{a_2 R}{\mu} 10^{-3} \cos 2 \left(\frac{360}{86400} t^* - 146°\right) \qquad (406)$$

$$\Delta S_{2,2} = - \frac{a_2 R}{\mu} 10^{-3} \sin 2 \left(\frac{360}{86400} t^* - 146°\right) \qquad (407)$$

$$\Delta C_{4,2} = - \frac{a_3 R}{\mu} 10^{-3} \cos 2 \left(\frac{360}{86400} t^* - 146°\right) \qquad (408)$$

$$\Delta S_{4,2} = + \frac{a_3 R}{\mu} 10^{-3} \sin 2 \left(\frac{360}{86400} t^* - 146°\right) \qquad (409)$$

CONTRIBUTIONS TO THE GRAVITATIONAL POTENTIAL EXPANSION COEFFICIENTS BY THE OCEAN TIDE

INPUT DATA

Concerned are the ocean tide modes M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m, and S_{sa}. For each of these tides, there exists a computer tape that lists the NP values of the tidal amplitude ξ_{ij} and the tidal phase δ_{ij}.

NP Number of grid points. This may be expected to be a five-digit integer.

ξ_{ij} Tidal amplitude on area element ij. If available in meters, use directly. Otherwise, convert to meters.

δ_{ij} Tidal phase angle on area element ij, in degrees.

Custodian of the tapes just mentioned is Ling Szeto of the Physical Sciences Software Branch. Also required are

R "Radius of the Earth," i.e., the semimajor axis of a suitable reference ellipsoid, in km. Trial value R = 6378.145 km.

ε^2 Square of eccentricity of reference ellipsoid. Trial value $\varepsilon^2 = 0.00669342$. In case it is desired to start from the ellipsoid flattening, f, find ε^2 from $\varepsilon^2 = (2 - f)f$.

μ Gravitational constant of the Earth, in km and s. Trial value $\mu = 398600.5\ km^3\ s^{-2}$

G Newton's constant, in km, kg, and s. Trial value $G = 6.6732E\text{-}20\ km^3\ kg^{-1}\ s^{-2}$.

ρ Density of water, in kg and km. Trial value $\rho = 1.E + 12\ kg\ km^{-3}$.

ρ^* Density of the ocean bottom, in kg and km. Trial value $\rho^* = 3.E + 12\ kg\ km^{-3}$.

σ Rate of mean longitude of Moon.

$$\sigma = \frac{180}{\pi} 1.40519E\text{-}04 \text{ deg s}^{-1}.$$

NMAX Limit on the number of terms in the expansion of the tide potential.

The following parameters Y,D, and t* specify the time instant at which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current time line of the orbit integration.

Y Number of the calendar year

D Number of the day within the year

t* Mean solar time at Greenwich (GMT, UTC), in s

ALGORITHM

Computer Routine for the Expansion Coefficients of the Tide Potential

The time-independent constituents of the expansion coefficients (${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, ${}^{\beta}H_{nm}$) of the tide potential will now be calculated from the amplitude and phase angle computer tapes, separately for each of the 11 ocean tide modes.

Once established, the coefficients ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, ${}^{\beta}H_{nm}$ will remain unchanged as long as the amplitude and phase tape for the particular tide mode remains valid. Thus, the computer routine under discussion will be exercised quite infrequently, namely just once each time a new edition of the just-mentioned amplitude and phase tape becomes available. That is expected to occur at intervals of several years, during which the present routine will remain dormant.

For each ocean tide mode execute the following algorithm. For the geometry associated with the index ij, see Figures 1, 2, and 3.

$$\alpha_{ij} = 10^{-3}(\rho - 0.0667\rho^*)G\Delta S_{ij}\xi_{ij}\cos\delta_{ij} \tag{501}$$

$$\beta_{ij} = 10^{-3}(\rho - 0.0667\rho^*)G\Delta S_{ij}\xi_{ij}\sin\delta_{ij} \tag{502}$$

$$\Delta S_{ij} = \left(\frac{\pi}{180}\right)^2 R^2 \sin\left(\frac{\pi}{180}\, j\right) \tag{503}$$

= area of the nonpolar surface area element for $j = 2,3,4,\cdots,j_{MAX} < 180$

$$(\Delta S_{ij})_{POLAR} = \frac{1}{2}\left(\frac{\pi}{180}\right)^3 R^2 \tag{504}$$

= area of the polar surface area element neighboring the North Pole. (Note that the southern polar surface area element is not needed because the computer tape mentioned on page 9 contains tide data for latitudes above 60 deg South only.)

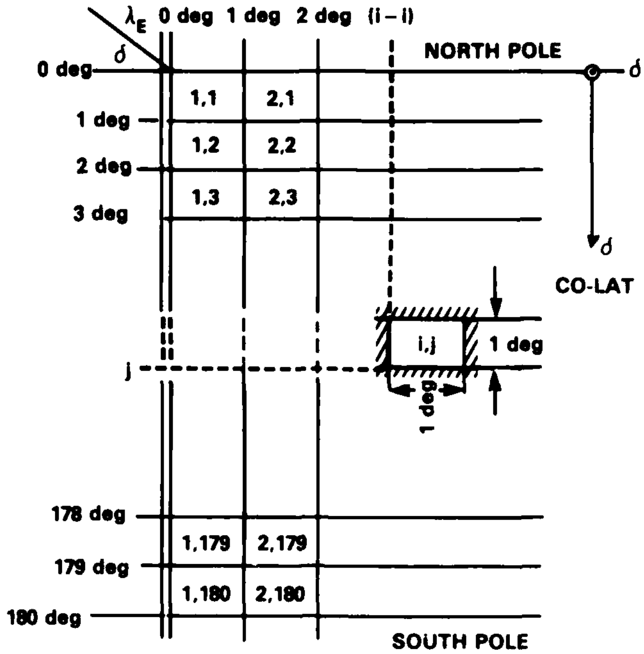

FIGURE 1. DIVISION OF THE EARTH'S SURFACE INTO AREA ELEMENTS ACCORDING TO THE SCHWIDERSKI OCEAN TIDE MODEL

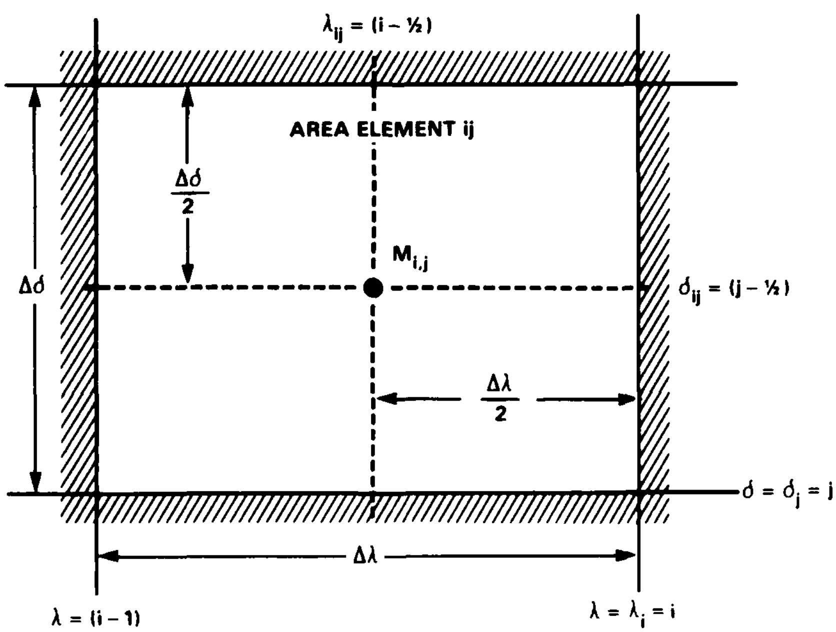

FIGURE 2. POSITION OF POINT MASS AT THE GEOMETRICAL CENTER OF THE SURFACE AREA ELEMENT ASSOCIATED WITH ij

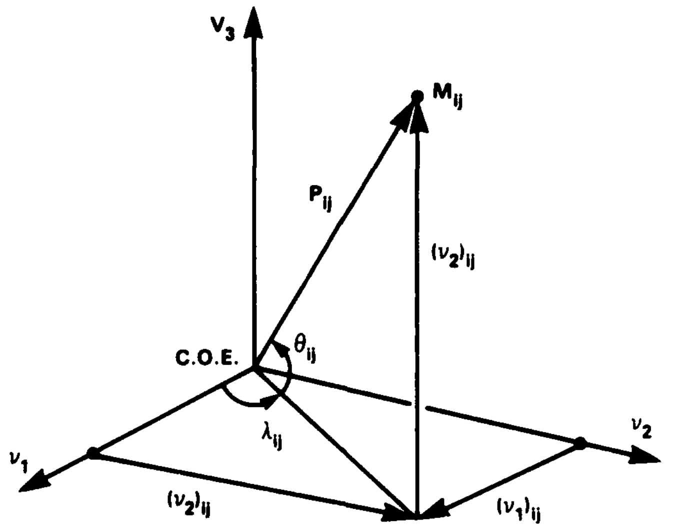

FIGURE 3. POSITION OF THE POINT MASS M_{ij} IN THE EARTH-FIXED CARTESIAN COORDINATE FRAME

$$\theta_{ij} = \frac{\pi}{2} - \frac{\pi}{180}\left(j - \frac{1}{2}\right) \text{ in rad} \tag{505}$$

$$\lambda_{ij} = \frac{\pi}{180}\left(i - \frac{1}{2}\right) \text{ in rad} \tag{506}$$

$$\rho_{ij} = \left(1 - \frac{\varepsilon^2}{2}\sin^2\theta_{ij}\right) \tag{507}$$

To each index ij assign now a counting number ν, $1 \leq \nu \leq NP$.

$$^{\alpha}F_{nm} = (2 - \delta_m^0)\frac{(n-m)!}{(n+m)!}\frac{1}{\mu R^{2n}}\sum_{\nu=1}^{NP}\rho_\nu^{2n+1}\ {}_{\alpha\nu}f_{nm}^{\nu} \tag{508}$$

$$^{\beta}F_{nm} = (2 - \delta_m^0)\frac{(n-m)!}{(n+m)!}\frac{1}{\mu R^{2n}}\sum_{\nu=1}^{NP}\rho_\nu^{2n+1}\ {}_{\beta\nu}f_{nm}^{\nu} \tag{509}$$

$$^{\alpha}H_{nm} = \frac{(n-m)!}{(n+m)!}\frac{1}{\mu R^{2n}}\sum_{\nu=1}^{NP}\rho_\nu^{2n+1}\ {}_{\alpha\nu}h_{nm}^{\nu} \tag{510}$$

$$^{\beta}H_{nm} = \frac{(n-m)!}{(n+m)!}\frac{1}{\mu R^{2n}}\sum_{\nu=1}^{NP}\rho_\nu^{2n+1}\ {}_{\beta\nu}h_{nm}^{\nu} \tag{511}$$

$$\delta_{\ell}^{k} = \begin{matrix} 1 \\ 0 \end{matrix} \text{ if } \begin{matrix} \ell = k \\ \ell \neq k \end{matrix} \qquad (512)$$

Calculate the f_{nm}^{ν} and h_{nm}^{ν} as follows, noting that

$$p_{\nu} = \frac{R}{\rho_{\nu}} \qquad (513)$$

and

$$h_{no}^{\nu} = 0 \text{ for all values of } n \qquad (514)$$

and that

$$h_{nm}^{\nu} = f_{nm}^{\nu} = 0 \text{ for any } n < m. \qquad (515)$$

Obtain, separately for each ν, the required f_{nm}^{ν} and h_{nm}^{ν} from the following recurrence relations:

To advance in n, evaluate

$$f_{n+1,m}^{\nu} = \frac{P_{\nu}}{n - m + 1} [(2n + 1) \sin \theta_{\nu} f_{n,m}^{\nu} - (n + m) p_{\nu} f_{n,m}^{\nu}] \qquad (516)$$

$$h_{n+1,m}^{\nu} = \frac{P_{\nu}}{n - m + 1} [(2n + 1) \sin \theta_{\nu} h_{n,m}^{\nu} - (n + m) p_{\nu} h_{n-1,m}^{\nu}] \qquad (517)$$

To advance in m, use

$$f_{n+1,n+1}^{\nu} = (2n + 1) p_{\nu} [\cos \theta_{\nu} \cos \lambda_{\nu} f_{n,n}^{\nu} - \cos \theta_{\nu} \sin \lambda_{\nu} h_{n,n}^{\nu}] \qquad (518)$$

$$h_{n+1,n+1}^{\nu} = (2n + 1) p_{\nu} [\cos \theta_{\nu} \cos \lambda_{\nu} h_{n,n}^{\nu} + \cos \theta_{\nu} \sin \lambda_{\nu} f_{n,n}^{\nu}] \qquad (519)$$

Start the recurrences from

$$f_{0,0}^{\nu} = \frac{1}{\rho_{\nu}} \qquad (520)$$

$$f_{1,0}^{\nu} = R \frac{\sin \theta_{\nu}}{\rho_{\nu}^{2}} \qquad (521)$$

$$h_{0,0}^{\nu} = h_{1,0}^{\nu} = 0 \qquad (522)$$

and terminate the procedure at n = NMAX.

Store the resulting ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$. They will be programmed as constant parameters into the computer routine for the perturbing potential and are expected to serve for all subsequent computer runs of that routine, until updated. Note that the ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$ are dimensionless quantities.

Calculation of the Astronomical Arguments for the Various Tide Components

Evaluate now, using double precision throughout, the Moon's longitude, s, the Sun's mean longitude, h, and the longitude of the Moon's mean perigee, p, at the beginning of the day UT in question

$$s_0 = s(T_0) \tag{523}$$

$$h_0 = h(T_0) \tag{524}$$

$$p_0 = p(T_0) \tag{525}$$

For each tide mode, find now the Astronomical Argument χ from

Tide Mode	χ in deg =	
M_2	$2(h_0 - s_0)$	(526)
S_2	0	(527)
N_2	$2h_0 - 3S_0 + p_0$	(528)
K_2	$2h_0$	(529)
K_1	$h_0 + 90$	(530)
O_1	$h_0 - 2S_0 - 90$	(531)
P_1	$-h_0 - 90$	(532)
Q_1	$h_0 - 3S_0 + p_0 - 90$	(533)
M_f	$2S_0$	(534)
M_m	$S_0 - p_0$	(535)
S_{sa}	$2h_0$	(536)

Note that the day number D, and thus χ, must be updated whenever the time argument t^* runs into the next day.

Computer Routine for the Increments to the Gravitational Potential Expansion Coefficients

For each tide mode, separately, use the above set of time-independent tide potential coefficients and the astronomical tide argument and calculate the time-independent expansion coefficients for the tide potential (note $n,m \leq$ NMAX). The latter are the increments to the gravitational potential expansion coefficients due to the particular ocean tide mode

$$\Delta C_{nm} = {}^{\alpha}F_{nm}\cos(\sigma t^* + \chi) + {}^{\beta}F_{nm}\sin(\sigma t^* + \chi) \tag{537}$$

$$\Delta S_{nm} = {}^{\alpha}H_{nm}\cos(\sigma t^* + \chi) + {}^{\beta}H_{nm}\sin(\sigma t^* + \chi) \tag{538}$$

The argument $(\sigma t^* + \chi)$ should be evaluated in double precision. The double precision sin and cos functions should be applied. Then calculate ΔF_{nm} and ΔH_{nm} and subsequently revert to single precision.

Note also that ΔF_{nm} and ΔH_{nm} are linear functions of the two trigonometric terms. To evaluate these trigonometric terms, let t_i^* and t_{i+1}^* be the values of Universal time for which subsequent integration steps are to be performed. To save computer time, update the trigonometric time factors as follows

$$\begin{aligned}\cos(\sigma t_{i+1}^* + \chi) &= \cos[(\sigma t_i^* + \chi) + \sigma\Delta t] \\ &= \cos(\sigma t_i^* + \chi)\cos \sigma\Delta t - \sin(\sigma t_i^* + \chi)\sin \sigma\Delta t\end{aligned} \tag{539}$$

$$\begin{aligned}\sin(\sigma t_{i+1}^* + \chi) &= \sin[\sigma t_i^* + \chi) + \sigma\Delta t] \\ &= \sin(\sigma t_i^* + \chi)\cos \sigma\Delta t + \cos(\sigma t_i^* + \chi)\sin \sigma\Delta t\end{aligned} \tag{540}$$

$$\Delta t = t_{i+1}^* - t_i^* \tag{541}$$

CONTRIBUTION TO THE GRAVITATIONAL POTENTIAL EXPANSION COEFFICIENTS BY THE SOLID EARTH TIDE

INPUT DATA

- Y year
- D number of day in year
- t^* Universal Time in s
- k_2 nominal second degree Love number. Default value $k_2 = 0.3$.
- R semimajor axis of reference ellipsoid in km. Default value $R = 6378.140$ km
- μ Earth's gravitational parameter. $\mu = 398600.5\ \mathrm{km^3\ s^{-2}}$
- μ_2 Moon's gravitational parameter. $\mu_2 = 4916.816\ \mathrm{km^3\ s^{-2}}$
- μ_3 Sun's gravitational parameter. $\mu_3 = 132.712\mathrm{E}{+}09\ \mathrm{km^3\ s^{-2}}$
- x_2^*, y_2^*, z_2^* the inertial, Cartesian coordinates of the Moon, in km, for the time instant (Y, D, t*), from the JPL Export Ephemeris. The reference frame is referred to equator and equinox corresponding either to Epoch J2000.0 or to Epoch 1950.0.
- x_3^*, y_3^*, z_3^* the inertial, Cartesian coordinates of the Sun, in km, for the time instant (Y, D, t*), from the JPL Export Ephemeris. The reference frame is referred to equator and equinox corresponding either to Epoch J2000.0 or to Epoch 1950.0.

ALGORITHM

The Earth-Fixed Moon and Sun Coordinates

Calculate now the Earth-fixed coordinates of the Moon (j = 2) and of the Sun (j = 3), x_j, y_j and z_j, at the time instant (Y, D, t*) from the corresponding inertial coordinates

$$\begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix} = (ABCD)_{Y,D,t^*} \begin{pmatrix} x_j^* \\ y_j^* \\ z_j^* \end{pmatrix} \tag{601}$$

where $(ABCD)_{Y,M,t^*}$ is the transformation matrix that manages the transition from the (inertial) ephemeris frame of reference to the Earth-fixed reference frame associated with the time instant (Y, D, t*). This transformation may be assumed to be a readily available subroutine of OMNIS.

Further calculate, for j = 2 and j = 3,

$$r_j = + \sqrt{x_j^2 + y_j^2 + z_j^2}$$

$$= + \sqrt{(x_j^*)^2 + (y_j^*)^2 + (z_j^*)^2} \tag{602}$$

$$\rho_j = + \sqrt{x_j^2 + y_j^2} \tag{603}$$

$$\cos \phi_j = \rho_j / r_j \tag{604}$$

$$\sin \phi_j = z_j / r_j \tag{605}$$

$$\sin 2\phi_j = 2 \sin \phi_j \cos \phi_j \tag{606}$$

$$\cos \lambda_j = x_j / \rho_j \tag{607}$$

$$\sin \lambda_j = y_j / \rho_j \tag{608}$$

$$\cos 2\lambda_j = \cos^2 \lambda_j - \sin^2 \lambda_j \tag{609}$$

$$\sin 2\lambda_j = 2 \sin \lambda_j \cos \lambda_j \tag{610}$$

where

r_j = distance from the geocenter to the Moon/Sun in km

ϕ_j = geocentric latitude of the Moon/Sun

λ_j = longitude (East) of the Moon/Sun

The Long-Period Tide Correction

Compute the increments for the normalized geopotential expansion coefficient $\bar{C}_{20}$ as follows.

$$\Delta\bar{C}_{20} = \frac{1}{\sqrt{5}} k_2 \frac{R^3}{\mu} \sum_{j=2}^{3} \frac{\mu_j}{(r_j)^3} P_{20}(\sin \phi_j) - \langle \Delta\bar{C}_{20} \rangle \tag{611}$$

$$P_{20}(x) = \frac{1}{2} (3 x^2 - 1) \tag{612}$$

The term $\langle \Delta\bar{C}_{20} \rangle$ is a constant reflecting the "Permanent Tide"

$$\langle \Delta\bar{C}_{20} \rangle = - 1.39119 \cdot 10^{-8} k_2 \tag{613}$$

It must be deleted when using $\bar{C}_{20}$ values derived by a method that already accounts for the effect of the Permanent Tide.

The Diurnal Tide Corrections

Calculate now the increments to the normalized geopotential expansion coefficients $\bar{C}_{21}$ and $\bar{S}_{21}$ ($n = 2$, $m = 1$, $n + m$ = odd)

$$\Delta\bar{C}_{21} = \frac{1}{3} \sqrt{\frac{3}{5}} k_2 \frac{R^3}{\mu} \sum_{j=2}^{3} \frac{\mu_j}{(r_j)^3} P_{21}(\sin \phi_j) \cos \lambda_j + \Delta\Delta\bar{C}_{21} \tag{614}$$

$$P_{21} (\sin \phi_j) = 3 \sqrt{1 - \sin^2 \phi_j} \sin \phi_j = \frac{3}{2} \sin 2\phi_j \tag{615}$$

$$\begin{aligned}
\Delta\Delta\bar{C}_{21} = & \sum_s (A_m \delta k_s H_s) \sin \theta_s \\
= & - 16.4 \cdot 10^{-12} \sin (\tau - s) \\
& - 49.6 \cdot 10^{-12} \sin (\tau + s - 2h) \\
& - 9.4 \cdot 10^{-12} \sin (\tau + s - N') \\
& + 507.4 \cdot 10^{-12} \sin (\tau + s) \\
& + 73.5 \cdot 10^{-12} \sin (\tau + s + N') \\
& - 15.2 \cdot 10^{-12} \sin (\tau + s + h - p_1) \qquad (616A) \\
= & \ 16.4 \cdot 10^{-12} \sin (\theta_g - 2F - 2\Omega) \\
& + 49.6 \cdot 10^{-12} \sin (\theta_g - 2F - 2\Omega + 2D) \\
& + 9.4 \cdot 10^{-12} \sin (\theta_g + \Omega) \\
& - 507.4 \cdot 10^{-12} \sin \theta_g \\
& - 73.5 \cdot 10^{-12} \sin (\theta_g - \Omega) \\
& + 15.2 \cdot 10^{-12} \sin (\theta_g + \ell') \qquad (616B)
\end{aligned}$$

$$\Delta\bar{S}_{21} = \frac{1}{3}\sqrt{\frac{3}{5}}\, k_2 \frac{R^3}{\mu} \sum_{j=2}^{3} \frac{\mu_j}{(r_j)^3} P_{21}(\sin \phi_j)\sin \lambda_j + \Delta\Delta\bar{S}_{21} \tag{617}$$

$$\begin{aligned}\Delta\Delta\bar{S}_{21} = {} & 16.4 \cdot 10^{-12} \cos (\theta_g - 2F - 2\Omega) \\ & + 49.6 \cdot 10^{-12} \cos (\theta_g - 2F - 2\Omega + 2D) \\ & + 9.4 \cdot 10^{-12} \cos (\theta_g + \Omega) \\ & - 507.4 \cdot 10^{-12} \cos \theta_g \\ & - 73.5 \cdot 10^{-12} \cos (\theta_g - \Omega) \\ & + 15.2 \cdot 10^{-12} \cos (\theta_g + \ell')\end{aligned} \tag{618}$$

The Semidiurnal Tide Corrections

Calculate finally the increments to the normalized geopotential expansion coefficients $\bar{C}_{22}$ and $\bar{S}_{22}$ ($n = 2$, $m = 2$, $n + m$ = even)

$$\Delta\bar{C}_{22} = \frac{1}{12}\sqrt{\frac{12}{5}}\, k_2 \frac{R^3}{\mu} \sum_{j=2}^{3} \frac{\mu_j}{(r_j)^3} P_{22}(\sin \theta_j)\cos 2\lambda_j + \Delta\Delta\bar{C}_{22} \tag{619}$$

$$P_{22}(\sin \phi_j) = 3 \cos^2 \phi_j \tag{620}$$

$$\begin{aligned}\Delta\Delta\bar{C}_{22} = {} & 39.5 \cdot 10^{-12} \cos 2(\theta_g - F - \Omega) \\ & + 18.4 \cdot 10^{-12} \cos 2(\theta_g - F - \Omega + D)\end{aligned} \tag{621}$$

$$\Delta\bar{S}_{22} = \frac{1}{12}\sqrt{\frac{12}{5}}\, k_2 \frac{R^3}{\mu} \sum_{j=2}^{3} \frac{\mu_j}{(r_j)^3} P_{22}(\sin \phi_j)\sin 2\lambda_j + \Delta\Delta\bar{S}_{22} \tag{622}$$

$$\begin{aligned}\Delta\Delta\bar{S}_{22} = {} & -39.5 \cdot 10^{-12} \sin 2(\theta_g - F - \Omega) \\ & - 18.4 \cdot 10^{-12} \sin 2(\theta_g - F - \Omega + D)\end{aligned} \tag{623}$$

The increments to the gravitational potential expansion coefficients, due to the solid Earth tide, are

$$\Delta C_{n,m} = \Delta\bar{C}_{n,m} \sqrt{\frac{(n - m)!(2n + 1)(2 - \delta_m^0)}{(n + m)!}} \tag{624}$$

$$\Delta S_{n,m} = \Delta \bar{S}_{n,m} \sqrt{\frac{(n - m)!(2n + 1)(2 - \delta_m^0)}{(n + m)!}} \tag{625}$$

$$\delta_m^0 \left\{ \begin{array}{lll} = 0 & & m \neq 0 \\ & \text{for} & \\ = 1 & & m = 0 \end{array} \right. \tag{626}$$

CONTRIBUTION TO THE GRAVITATIONAL POTENTIAL EXPANSION COEFFICIENTS BY THE SOLID EARTH TIDE (TERRA/CELEST FORMULATION--BACKUP FORMULATION FOR OMNIS)

INPUT DATA

Y year

D number of day in year

t^* Universal Time in s

k_2 nominal second degree Love number. Default value $k_2 = 0.3$.

R semimajor axis of reference ellipsoid in km. Default value $R = 6378.140$ km.

μ Earth's gravitational parameter. $\mu = 398600.5 \text{ km}^3 \text{ s}^{-2}$.

μ_2 Moon's gravitational parameter. $\mu_2 = 4916.816 \text{ km}^3 \text{ s}^{-2}$.

μ_3 Sun's gravitational parameter. $\mu_3 = 132.712\text{E}+09 \text{ km}^3 \text{ s}^{-2}$.

x_2^*, y_2^*, z_2^* the inertial, Cartesian coordinates of the Moon, in km, for the time instant (Y, D, t*), from the JPL Export Ephemeris. The reference frame is referred to equator and equinox corresponding either to Epoch J2000.0 or to Epoch 1950.0.

x_3^*, x_3^*, x_3^* the inertial, Cartesian coordinates of the Sun, in km, for the time instant (Y, D, t*), from the JPL Export Ephemeris. The reference frame is referred to equator and equinox corresponding either to Epoch J2000.0 or to Epoch 1950.0.

ALGORITHM

The Earth-Fixed Moon and Sun Coordinates

Calculate now the Earth-fixed coordinates of the Moon ($j = 2$) and of the Sun ($j = 3$), x_j, y_j and z_j, at the time instant (Y, D, t^*) from the corresponding inertial coordinates

$$\begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix} = (ABDC)_{Y,D,t^*} \begin{pmatrix} x_j^* \\ y_j^* \\ z_j^* \end{pmatrix} \tag{701}$$

where $(ABCD)_{Y,D,t^*}$ is the transformation matrix that manages the transition from the (inertial) ephemeris frame of reference to the Earth-fixed reference frame associated with the time instant (Y, D, t^*). This transformation may be assumed to be a readily available subroutine of OMNIS.

Further calculate, for $j = 2$ and $j = 3$

$$r_j = +\sqrt{x_j^2 + y_j^2 + z_j^2}$$

$$= +\sqrt{(x_j^*)^2 + (y_j^*)^2 + (z_j^*)^2} \tag{702}$$

$$\rho_j = +\sqrt{x_j^2 + y_j^2} \tag{703}$$

$$\cos \phi_j = \rho_j / r_j \tag{704}$$

$$\sin \phi_j = z_j / r_j \tag{705}$$

$$\sin 2\phi_j = 2 \sin \phi_j \cos \phi_j \tag{706}$$

$$\cos \lambda_j = x_j / \rho_j \tag{707}$$

$$\sin \lambda_j = y_j / \rho_j \tag{708}$$

$$\cos 2\lambda_j = \cos^2\lambda - \sin^2\lambda \tag{709}$$

$$\sin 2\lambda_j = 2 \sin \lambda \cos \lambda \tag{710}$$

where

r_j = distance from the geocenter to the Moon/Sun in km

ϕ_j = geocentric latitude of the Moon/Sun

λ_j = longitude (East) of the Moon/Sun

The Increments to the Potential Expansion Coefficients

The increments to the gravitational potential expansion coefficients, due to the solid Earth tide (TERRA/CELEST Version), are

$$\Delta C_{2,0} = k_2 \sum_{j=2}^{3} \frac{\mu_j}{\mu} \frac{R^3}{r_j^3} P_2(\sin \phi_j) \tag{711}$$

$$\Delta C_{2,1} = k_2 \sum_{j=2}^{3} \frac{1}{3} \frac{\mu_j}{\mu} \frac{R^3}{r_j^3} P_2^1(\sin \phi_j)\cos \lambda_j \tag{712}$$

$$\Delta S_{2,1} = k_2 \sum_{j=2}^{3} \frac{1}{3} \frac{\mu_j}{\mu} \frac{R^3}{r_j^3} P_2^1(\sin \phi_j)\sin \lambda_j \tag{713}$$

$$\Delta C_{2,2} = k_2 \sum_{j=2}^{3} \frac{1}{12} \frac{\mu_j}{\mu} \frac{R^3}{r_j^3} P_2^2(\sin \phi_j)\cos 2\lambda_j \tag{714}$$

$$\Delta S_{2,2} = k_2 \sum_{j=2}^{3} \frac{1}{12} \frac{\mu_j}{\mu} \frac{R^3}{r_j^3} P_2^2(\sin \phi_j)\sin 2\lambda_j \tag{715}$$

where

$$P_2 (\sin \phi_j) \equiv \frac{1}{2}(3 \sin^2 \phi_j - 1) \tag{716}$$

$$P_2^1 (\sin \phi_j) \equiv 3\sqrt{1 - \sin^2 \phi_j} \sin \phi_j \equiv \frac{3}{2} \sin 2\phi_j \tag{717}$$

$$P_2^2 (\sin \phi_j) \equiv 3 \cos^2 \phi_j \tag{718}$$

Note that the present algorithm is a special case of the algorithm for the contribution to the gravitational expansion coefficients by the solid Earth tides on page 16 (Wahr theory terms deleted).

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